

Geometric CP Violation with Extra Dimensions

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Abstract

We discuss how CP symmetry can be broken geometrically through orbifold construction in hidden extra dimensions in the context of D-brane models for particle unifications. We present a few toy models to illustrate the idea and suggest ways to incorporate this technique in the context of realistic models.

I. INTRODUCTION

Origin of discrete symmetry violations observed in nature, such as parity or CP violations, is still a mystery. In the standard model of Glashow, Weinberg and Salam, parity violation was put in “by hand” by excluding the right handed neutrinos from the theory. Similarly in the three generation Kobayashi-Maskawa extension, the lone CP violating phase is also an input that arose by making the Yukawa couplings complex and no insight is gained as to how nature broke CP symmetry [1].

In early 1970’s, it was pointed out that if one assumed the standard model to be a part of the left-right symmetric model of weak interactions, a more satisfactory framework for the origin of parity violation can be obtained [2]. The recent discovery of neutrino masses and their understanding in terms of the seesaw mechanism may be construed as a sign pointing towards ultimate left-right symmetry of Nature. The question one must then address is how left-right symmetry is broken in Nature.

Ever since Einstein’s general theory of relativity, physicist have often tried to see if an idea or a theory can be realized geometrically. The door to this possibility has been open more widely by the realization that the Nature can accommodate extra dimensions of spacetime as long as they are sufficiently hidden. This has received strong theoretical

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support from superstring theories which require the existence of extra hidden dimensions for their consistency. In this context, an intriguing mechanism for explaining the existence of broken symmetries is to connect them with the existence of hidden compact dimensions in Nature. Some examples realizing this for parity were suggested in Ref. [3]. In fact it is tempting to speculate, whether all symmetry violations in Nature owe their origin to the presence of extra dimensions. Recently, it has been suggested that weak gauge symmetry [4] as well as the grand unification symmetries [5] could be broken by the extra dimensional effects.

In this brief note, we explore the possibility that CP symmetry breaking is connected with the existence of hidden compact dimensions in Nature. We present several simple examples where the CP symmetry is broken by orbifold projections. We then suggest ways to incorporate them into more realistic models.

II. A TOY MODEL

Consider a simple $U(1)$ gauge model with four left-handed fermions, $f_{1,3}$ with charge +1 and $f_{2,4}$ with charge -1 and one real scalar η which is $U(1)$ neutral. We will use flat fifth dimension so that when the fifth dimension y is compactified, we can use sine and cosine expansion. All the fermion fields are assumed to be bulk fields (with four 4-dimensional chiral fermions combined into two 5-dimensional fermions) and the η field is assumed to be located in the brane at $y = 0$. Let us assume that the fifth dimensional space is projective and therefore the Lagrangian is required to be invariant under $y \rightarrow -y$ tranformation and all the bulk fields tranformation under the even or odd representation of an assoctated Z_2 symmetry. We shall assume the brane fields (including η) to be even under Z_2 . As far as the bulk fermions go, invariance of the bulk kinetic energy term $\bar{f}_{iL}\partial_y f_{iR}$ under Z_2 implies that f_{iL} and f_{iR} have opposite Z_2 . This means that if one of them is odd under Z_2 , the other is even. Let us assume that f_{1L}, f_{2L} are Z_2 even while f_{3L}, f_{4L} are Z_2 odd.

The 5-dimensional Lagrangian for this model relevant for our discussion can be written as:

$$\mathcal{L}(x, y) = \frac{1}{M_*} \delta(y) \left[\lambda \eta (f_{1L}^T C^{-1} f_{2L} - (f_{3L})^c C^{-1} (f_{4L})^c) \right] + \mu (f_{1L}^T C^{-1} f_{2L} + (f_{3L})^c C^{-1} (f_{4L})^c) + H.c. \quad (1)$$

where $(f_L)^c$ is the usual 4-dimensional charge conjugation of f_L . The above coupling structure can obviously be maintained naturally through some flavor symmetry ($U(1) \times U(1)$ for example) for the fermions f_i which we shall not elaborate. Under CP symmetry, we define the fields to transform as:

$$\begin{aligned} f_{1L} &\rightarrow (f_{3L})^c \\ f_{2L} &\rightarrow (f_{4L})^c \\ \eta &\rightarrow -\eta \end{aligned} \quad (2)$$

where we have defined parity as the inversion of only the three familiar space coordinates. Suppose we now assume that f_{1L} and f_{2L} are even and $f_{3L,4L}$ are odd under Z_2 , then on the brane located at $y = 0$, the Fourier expansion of the Z_2 even fields involve only the cosine

modes (i.e. $\cos\frac{\pi ny}{R}$) and the Z_2 odd fields involve only the sine modes (i.e. $\sin\frac{n\pi y}{R}$). As a result, the fields $f_{3L,4L}$ vanish on the standard model brane. Clearly this results in a spectrum of fields on the observed 3-brane, which is asymmetric with respect to the $f_{1,2}$ versus $f_{3,4}$. This leads to breakdown of CP invariance, since under CP, $f_{1,2} \rightarrow f_{3,4}^c$ respectively. Note that if we had chosen same boundary conditions for all f_i fields, CP would have remained an unbroken symmetry on the 3-brane at $y = 0$.

As a result of the asymmetric boundary conditions, all modes of the fermion fields f_i will appear in the 4-dimensional Lagrangian symmetrically except for the lowest modes. The resulting low energy Lagrangian involving only the zero modes can be written as:

$$\lambda \eta f_{1L}^{0T} C^{-1} f_{2L}^0 + \mu f_{1L}^{0T} C^{-1} f_{2L}^0 + h.c. \quad (3)$$

which is clearly CP violating through the phase $Im(\lambda^* \mu)$. (One can of course pick a phase convention to make μ real).

Let us assume that the η field (the “messenger” field) couple to the standard model fermions in the brane such as an electron. To illustrate that CP is originally conserved in the sector of visible fields in the brane but broken after the asymmetric orbifold conditions are enforced, one can couple η to the electron as $i\eta\bar{e}\gamma_5 e$, note that γ_5 is needed here for CP symmetry. Then, assuming that all fermions are charged, one generate an edm for electron via the two loop diagram in Fig. 1 [6] with f_1, f_2 in the loop. But if the (f_3, f_4) fermions were also allowed in the loop, as would be the case if we were to impose the symmetric orbifold boundary condition for them, it will cancel the contribution of (f_1, f_2) and would result in zero edm for electron.

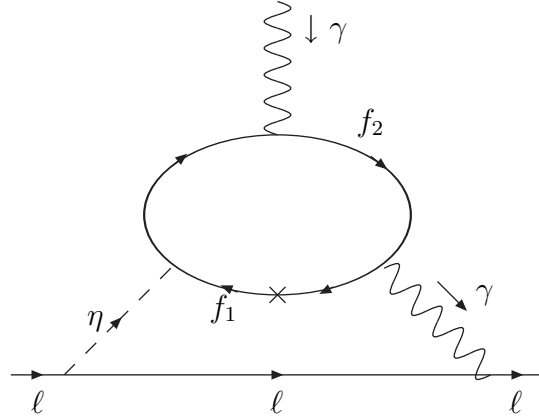


Fig. 1 The two-loop graph that contributes to edm of lepton ℓ . The cross location denotes a possible mass insertion. In the fermion loop, if the orbifold boundary conditions break CP symmetry, the dominant contribution comes from the zero modes of the $f_{1,2}$ and leads to nonzero edm. On the other hand, if the orbifold boundary conditions donot break CP, all the KK modes are symmetric between $f_{1,2}$ and $f_{3,4}$ and due to the negative sign in the Yukawa couplings, they cancel each other.

Note that, even without CP violating projection, at one loop level, one can also generate operators $(f_1)^{cT} C^{-1} \sigma_{\mu\nu} \gamma_5 f_2 F^{\mu\nu}$ and $(f_3)^{cT} C^{-1} \sigma_{\mu\nu} \gamma_5 f_4 F^{\mu\nu}$ with equal coefficient (proportional to $Im(\lambda^2 \mu^*)$). These are usually refered to as the electric dipole moment operators

of the chiral pairs. However, their existence do not imply CP violation. Under CP transformation, $(f_1)^{cT} \sigma_{\mu\nu} \gamma_5 f_2 F^{\mu\nu}$ operator is mapped into $(f_3)^{cT} \sigma_{\mu\nu} \gamma_5 f_4 F^{\mu\nu}$ and therefore CP conservation implies the two have to have the same coefficient. This slightly counter-intuitive situation arises purely because the system has degenerate degrees of freedom which makes a more general definition of CP possible.

Note that with the coupling to the electron, the theory is CP violating even for vanishing $Im(\lambda^* \mu)$. This is because the coupling to electron defines η as CP odd, as originally imposed, in the mean time, the coupling to $f_{1,2}$ in Eq.(3) is consistent with this CP definition only for the special case that λ is purely imaginary. Therefore for generic λ CP is broken.

Another independent manifestation of how the orbifold condition can lead to CP violation on the brane is to note that CP invariance for the (η, f_i) sector implies that if we choose a potential for the η field as $V(\eta) = m_\eta^2 \eta^2 + \lambda_\eta \eta^4$, with $m_\eta^2 > 0$, then $\langle \eta \rangle = 0$. Thus vacuum also leaves CP as a good symmetry. Now if we take one loop effects with symmetric boundary conditions for all f_i 's, then the tadpole diagrams will cancel between $f_{1,2}$ and $f_{3,4}$ keeping the $\langle \eta \rangle = 0$ vev stable under radiative corrections. However once we impose asymmetric boundary conditions between $f_{1,2}$ and $f_{3,4}$, then there will be a nonvanishing tadpole contribution leading to a vev of the η field and one will have spontaneous breakdown of CP invariance.

As the second example, one can choose to use bulk scalar instead of fermions f_i to implement CP violating projection. For example, one can replace f_i by a pair of bulk complex charged bosons, b_1, b_2 which transform into each other under CP symmetry.

$$\lambda \eta (b_1^* b_1 - b_2^* b_2) + m^2 (b_1^* b_1 + b_2^* b_2) + i h \eta \bar{e} \gamma_5 e \quad (4)$$

where λ and h are real couplings. Note that b_i are degenerate in mass as required by CP symmetry. Just as in the case of bulk fermion model, if CP were not broken, the edm of electron through the two loop diagram (Fig. 1) with an inner bosonic loop would be zero due to the cancellation between the contributions from the two bosons as explicit calculation in Ref. [7] had shown. However, if CP is broken by the orbifold construction, non-vanish edm for electron results as expected.

A common character of all the examples is that, in higher dimensions, there is a degeneracy in the spectrum of the bulk fields such that a more general definition of CP is possible. The degenerate fields can be either fermions or bosons. The resulting four dimensional theory on the brane can have either soft or hard CP violation. While we arrange the CP symmetry in higher dimension by hand in our examples, in a realistic unified theory, this symmetry may arise automatically or accidentally because such a higher energy theory, such as the superstring theory, naturally has smaller number of coupling constants and larger symmetry which makes particle spectral degeneracy more likely and thus leaves room for a broader, less conventional, definition of CP symmetry. It is also true that CP violating projective condition is arranged by hand in our mechanism as in any other CP violating mechanism in the literature, however, we believe this is the first time that it is implemented geometrically. Since compactification is unavoidable in higher dimensional theory, it should not be surprising to see CP broken in the process given that a natural mechanism exists.

We should also emphasize that, the usual common sense, initiated by Landau, that the CP violation is tightly related to physical complex coupling constants is true only for systems without spectral degeneracy. For example, in our toy model in Eq.(1), there is clearly a

physical complex coupling constant in the relative phase between λ and μ . However, before one imposes the CP violating projective condition, this complex phase does not give rise to CP violation. Spectral degeneracy makes it possible to define a CP symmetry even though the Lagrangian contains physical complex couplings.

III. TOWARDS A REALISTIC EXAMPLE

In this section, we show how the idea of the previous section can be used to generate a multi-Higgs [8] model of CP violation. Although this model is presently very highly constrained by experiments, because it predicts too large a value for the neutron electric dipole moment [9], $b \rightarrow s + \gamma$ etc., we choose to discuss this since it provides a very straight forward way to illustrate how our idea can generate a realistic model of CP violation.

As before, consider four fermion fields $f_{1,2}$ and $f_{3,4}$ in the bulk and a singlet field η in the brane. Under CP, we assume the same transformation rules as before and as we saw once the orbifold conditions asymmetrize the fermion spectrum, the η field acquires a nonzero vev. The question now is how does one transmit it to the standard model.

For this purpose, let us now assume a multi-Higgs extension of the standard model (gauge fields, fermions as well as Higgses) living in the brane. We assume the model to be CP invariant under the usual definition of CP transformation of all the fields. We couple the η field to the brane fields in a way that preserves CP invariance. For instance, for a three Higgs doublet model, one can choose, the following renormalizable Higgs potential:

$$\begin{aligned}
 V(\phi_a, \eta) = & V_0(\phi_a^\dagger \phi_a) + \sum_{a,b} \mu_{ab}^2 (\phi_a^\dagger \phi_b + \phi_b^\dagger \phi_a) \\
 & + i\mu'_{ab} \eta (\phi_a^\dagger \phi_b - \phi_b^\dagger \phi_a) \\
 & + \lambda_{abcd} \phi_a^\dagger \phi_b \phi_c^\dagger \phi_d,
 \end{aligned} \tag{5}$$

with appropriate discrete symmetry to suppress flavor changing neutral current. Note now that once η acquires a vacuum expectation value (vev), there is a CP phase in the Higgs sector that will lead to complex vev for the Higgs fields and hence to the well known Weinberg profile for CP violation. We shall leave the discussion of other ways of generating a realistic model to a future publication.

IV. PROFILE OF GEOMETRIC CP VIOLATION IN MSSM

In this section, we apply this new mechanism to generate CP violation in the minimal supersymmetric standard model (MSSM) to provide an example of how it can be implemented in other realistic models. For this purpose, we start with the usual MSSM field content in the brane (i.e. $SU(2)_L \times U(1)_Y$ gauge group and superfields $Q, L, u^c, d^c, e^c, H_u, H_d$) augmented by the inclusion of a single superfield, which will be the “messenger” of CP violation. In the bulk we will now have N=2 supersymmetry. We will have two N=2 hypermultiplets in the brane, denoted by its N=1 components $(H_1, H_1^c; H_2, H_2^c)$. Under CP symmetry, we assume the MSSM fields to transform as usual i.e. $Q \rightarrow Q^*$, etc. The rest of the fields transform as follows:

$$\begin{aligned}
\eta &\rightarrow -\eta^* \\
H_1 &\rightarrow H_2^{c*} \\
H_2 &\rightarrow H_1^{c*}
\end{aligned}
\tag{6}$$

We assume the theory prior to compactification to be CP symmetric so that the only phase in the theory is in the coupling of the η fields to the bulk fields $H_{1,2}$:

$$\begin{aligned}
W_\eta = \eta(\lambda H_1 H_2 - \lambda^* H_1^c H_2^c) + M_1 \eta^2 \\
+ M_2 (H_1 H_2 + H_1^c H_2^c)
\end{aligned}
\tag{7}$$

where $M_{1,2}$ are masses expected to be of order of the fundamental scale of the theory. Now note that since the bulk kinetic energy leads to a term of the form [4] $H \partial_y H^c$, the required condition for CP violation i.e. H and H^c have opposite Z_2 parity is automatically satisfied and CP violation in the brane will ensue rather naturally due to asymmetric spectrum of the bulk fields.

To see the profile of CP violation, let us write down the superpotential in the brane involving the η fields (the usual MSSM superpotential terms involving the MSSM fields are omitted for simplicity). To incorporate supersymmetry breaking, we have the usual hidden sector mechanisms in mind. We will use a singlet field S to implement the susy breaking by choosing $\langle F_S \rangle = M^2 \approx (10^{11})^2 \text{ GeV}^2$.

$$W_{brane} = (i\eta + M_{wk})(a + b \frac{S}{M_{Pl}}) H_u H_d
\tag{8}$$

We have not written terms that are suppressed by higher powers of M_{Pl} since their effect on CP violation is negligible.

CP violation in the MSSM arises when the field η acquires a nonzero vev via the tadpole diagrams involving $H_{1,2}$ fields. In the supersymmetric limit, due to the nonrenormalization theorem of supersymmetry, $\langle \eta \rangle = 0$. It is then easy to see that $\langle \eta \rangle \simeq \frac{M_{susy}}{16\pi^2}$, where M_{susy} is the usual scale of superpartner masses. This leads to a profile of MSSM CP violation where the only CP violating terms are the μ and the $B\mu$ terms. Furthermore, the CP phase can be naturally of order 10^{-2} due to the presence of the factor $16\pi^2$ above. There is no CP phase of the usual KM type. The detailed phenomenological implications of this model will be the subject of a future publication. However, we shall emphasize that this model only provides an example of how the mechanism can provide interesting CP violating physics. It is in no way unique.

V. CONCLUSION

In this brief note we have pointed out a novel mechanism for breaking CP symmetry in a geometric way using the extra compact dimensions. The essential idea is that the asymmetrization of the spectrum by orbifold conditions can lead to CP violating effects. Note that this is very different from many recent papers [10] on CP violation in models with extra dimensions in which CP violation is put into either a Higgs VEV on some other brane or a susy breaking VEV's. In our case, the mechanism is genuinely geometrical in nature. We have also illustrated how this mechanism can be used to generate realistic models of CP

violation in the standard model as well as MSSM. In a future theory of everything, such as the string theory, it is likely that the theory will be so constrained that it leaves no room for CP violation at the fundamental level. In that case it is interesting to entertain the idea that CP violation arises out of the “twisting and turning” of the compactified extra space en route to producing the four dimensional world we live in.

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